RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2019

FIRST YEAR [BATCH 2019-22]

Date : 16/12/2019 **MATHEMATICS (General)**

Time : 11 am – 2 pm **Paper : I** Full Marks : 75

(Use a separate Answer book for each group)

Group – A

Answer **any five** questions of the following : [5 × 5]

1. The equation $3x^2+2xy+3y^2-18x-22y+50=0$ is reduced to $4x^2+2y^2=1$, when referred to rectangular axes through the point (2,3). Find the inclination of the latter axes to the former. (5)

2. Find the equation of the bisectors of angle between the lines $x^2-5xy+4y^2+x+2y-2=0$. [5]

3. Find the equation of the common tangent to parabolas $y^2=32x \text{ and } x^2=108y$. [5]

4. Show that if one of the lines given by the equation $ax^2+2hxy+by^2=0$ be perpendicular to one of the lines given by $a'x^2+2h'xy+b'y^2=0$ then $(aa'-bb')^2+4(ah'+hb')(ha'+bh')=0$. [5]

5. Reduce the equation $5x^2-6xy+5y^2-4x-4y-4=0$ to its cannonical form. Hence find the nature of the curve. [5]

6. The gradient of one of the straight lines of $ax^2+2hxy+by^2=0$ is twice that of the other. Show that $8h^2=9ab$. [5]

7. Deduce the equation of the director circle of the ellipse $\frac{x^2}{9}+\frac{y^2}{3}=16$. [5]

8. If PA and PB be the two tangents to the conic $\frac{l}{r}=1-e\cos\theta$ at $\alpha \text{ and } \beta$ respectively, then show that PS (S is the focus) bisects the angle ASB. [5]

Group – B

Answer **any five** questions of the follwing : [5 × 10]

9. a) If $x+\frac{1}{x}=2\cos\frac{\pi}{7}$, then show that $x^7+\frac{1}{x^7}=-2$. [5]

   b) If $\cosh^{-1}(x+iy)+\cosh^{-1}(x-iy)=\cosh^{-1}b$, where x, y, b are real and b>1. Prove that the point (x,y) lies on an ellipse. [5]

10. a) Show that the roots of the equation $\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}=\frac{1}{x}\ (a>b>c>0)$, are all real. [5]

    b) If $x^4+4x^3-8x^2+k=0$ has 4 unequal roots, prove that 0<k<3. [5]

11. (a) Do the vectors (1, 1, 0) , (1 , 0 , 1) , (0, 1, 1) form a basis for the real vector space $\mathbb{R}^3$? Justify. [5]

(b) Prove that $S=\{(x,y,z): x-3y+4z=0\}$ is a subspace of $\mathbb{R}^3$. [5]

12. (a) Define eigen values of a square matrix. State Caley-Hamilton theorem. [3+2]

(b) Find the eigen values and eigen vectors for $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. [5]

13. (a) Show that $S=\{(x,y,z): x+y+z=0=2x-3y+z\}$ is a subspace of $\mathbb{R}^3$. [5]

(b) α, β, γ are the roots of the equation $x^3-12x+65=0$. Find the equation whose roots are $(\beta-\gamma)^2, (\gamma-\alpha)^2, (\alpha-\beta)^2$. [5]

14. (a) Prove that cube root of unity with multiplication as binary operation form a group.

(b) If $(G,*)$ be a finite group of even order, prove that G contains an odd number of elements of order 2.

(c) Prove that $\mathbb{R}^+=\{x\in\mathbb{R} \mid x>0\}$ is a subgroup of the multiplicative group $\mathbb{R}^*=\{x\in\mathbb{R} \mid x\neq 0\}$.

[2+5+3]

15. (a) Let $\mathbb{Q}'=\mathbb{Q}\setminus\{1\}$. Prove that $\mathbb{Q}'$ is an abelian group with respect to '$*$', defined by $a*b=a+b-ab;\ a,b\in\mathbb{Q}'$.

(b) Let G be an abelian group. Prove that the subset $H=\{g\in G \mid g=g^{-1}\}$ forms a subgroup of G.

[5+5]

16. (a) Show that the ring of matrices $\left(\begin{bmatrix} 2a & 0 \\ 0 & 2b \end{bmatrix} \mid a,b\in\mathbb{Z}\right)$ contains divisor of zero.

(b) Show that the set $S=\{a+b\sqrt{2} \mid a,b\, are\, rationals\}$ forms a field with respect to addition and multiplication.

(5+5)

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